Published by Institute of Physics Publishing for SISSA/ISAS
Received: *June 9, 2004*
Accepted: *June 10, 2004*

No-scale $D = 5$ supergravity from Scherk-Schwarz reduction of $D = 6$ theories

Laura Andrianopoli[a] **and Sergio Ferrara**[ab]
[a] *Department of Physics, Theory Division, CERN*
CH 1211 Geneva 23, Switzerland
[b] *INFN, Laboratori Nazionali di Frascati*
Italy
E-mail: Laura.Andrianopoli@cern.ch, Sergio.Ferrara@cern.ch

Maria A. Lledó
Departament de Física Teòrica, Universitat de València and IFIC
C/Dr. Moliner, 50, E-46100 Burjassot (València), Spain
E-mail: Maria.Lledo@ific.uv.es

Abstract: We perform a generalized dimensional reduction of six dimensional supergravity theories to five dimensions. We consider the minimal $(2, 0)$ and the maximal $(4, 4)$ theories. In each case the reduction allows us to obtain gauged supergravities of no-scale type in dimension five with gauge groups that escape previous classifications. In the minimal case, the geometric data of the reduced theory correspond to particular cases of the $D = 5$ real special geometry. In the maximal case we find a four parameter solution which allows partial breaking of supersymmetry.

Keywords: Supersymmetry Breaking, Supergravity Models.

Contents

1. Introduction

In the present paper we discuss the Scherk-Schwarz (SS) dimensional reduction [1] on S^1 of $D = 6$ ungauged supergravity theories with 8 and 32 supercharges. The reduction gives supergravities in five dimensions with a flat gauge group. Such flat gaugings appear in four dimensions in the context of both, SS [2]–[5] and flux compactifications [6].

The SS mechanism relies on the presence of a global symmetry group of the higher dimensional theory. The class of no-scale supergravities at $D = 5$ that we obtain depend then on the global symmetry of the $D = 6$ theory [7].

In the $(2, 0)$ (minimal) theories [7]–[9] there are three kinds of matter multiplets: the vector multiplet which has no scalars, the tensor multiplet with 1 scalar and the hypermultiplet with four scalars. If we have n_T tensor multiplets and n_H hypermultiplets, the scalar manifold is a product

$$\frac{\mathrm{SO}(1, n_T)}{\mathrm{SO}(n_T)} \times \mathcal{M}_Q \,, \tag{1.1}$$

where $\mathcal{M}_Q$ is a quaternionic manifold of quaternionic dimension n_H [10]. The SS phase is, in general, a combination of isometries of both manifolds.

The graviton multiplet contains a self dual tensor field, while the tensors from the tensor multiplets are anti-self dual. We denote the set of tensor fields as B^r, $r = 0, \ldots n_T$, with B^0 pertaining to the graviton multiplet.

When vector multiplets are present, the vectors (A^x, $x = 1, \ldots n_V$) couple to the tensor fields and their interaction term is of the form [11]–[14]

$$C_{rxy} B^r \wedge F^x \wedge F^y \,, \qquad F^x = dA^x \,,$$

with C_{rxy}=constant. This term is related by supersymmetry to the kinetic term of the vectors

$$C_{rxy} b^r F^x \wedge {}^* F^y \,. \tag{1.2}$$

The fields b^r, $r = 0, \dots n_T$ satisfy the constraint

$$\eta_{rs} b^r b^s = 1\,,$$

which defines the manifold $\mathrm{SO}(1, n_T)/\mathrm{SO}(n_T)$. The terms (1.2) explicitly break the $\mathrm{SO}(1, n_T)$ symmetry, unless the vector fields A^x transform under some n_V-dimensional representation R_V of $\mathrm{SO}(1, n_T)$ with the property that $\mathrm{Sym}(R_V \otimes R_V)$ contains the vector representation. In that case, the constants C_{rxy} can be chosen as invariant couplings. This happens, for instance, if R_V is a spinor representation of $\mathrm{SO}(1, n_T)$. Remarkably, this choice leads after dimensional reduction on S^1 to the real special geometries which are homogeneous (in particular, symmetric) spaces [15]–[18].

Under this assumption, the SS reduction produces a theory with a flat gauge group of the form $\mathrm{U}(1) \ltimes R_V$, where the U(1) generator is in the Cartan subalgebra (CSA) of the maximal compact subgroup $\mathrm{SO}(n_T)$ of the global symmetry $\mathrm{SO}(1, n_T)$. The U(1) group is gauged by the vector coming from the metric in dimension six. The tensors are in a vector representation of $\mathrm{SO}(1, n_T)$, so they are charged under U(1) (except for some singlets as B^0).

We remark that in order to introduce a SS phase in the tensor-vector multiplet sector it is actually sufficient that the constants C_{rxy} preserve a U(1) subgroup of $\mathrm{SO}(1, n_T)$, which is a much weaker assumption. The examples that we will consider in this paper have the full $\mathrm{SO}(1, n_T)$ symmetry.

The generator of the group U(1) may also have a component on the isometries of the quaternionic manifold [19]; in particular, it may have a component in the CSA of the SU(2) R-symmetry, then breaking supersymmetry (notice that this can happen even if hypermultiplets are not present, corresponding to a $D = 5$ Fayet-Iliopoulos term). The SS reduction leads to a positive semidefinite potential also in this case. The $D = 5$ interpretation of the theory must correspond to a gauging with the term $V_R = 0$ (see section 2.2 and ref. [20]).

In the case of (maximal) (4,4) $D = 6$ supergravity [21], the sigma model is

$$\frac{G}{H} = \frac{\mathrm{SO}(5,5)}{\mathrm{SO}(5) \times \mathrm{SO}(5)}\,.$$

The SS phase lies in the CSA of $\mathrm{USp}(4) \times \mathrm{USp}(4)$ ($\mathrm{USp}(4) = \mathrm{Spin}(5)$) so that the theory contains $4 = \mathrm{rank}(\mathrm{USp}(4) \times \mathrm{USp}(4))$ mass parameters. There are 16 vectors in six dimensions corresponding to the chiral (real) spinor representation of $\mathrm{Spin}(5,5)$, so the flat group is $\mathrm{U}(1) \ltimes \mathbb{R}^{16}$ [22]. This is a straightforward generalization of the $D = 4$, $N = 8$ case studied in ref. [2]. Partial supersymmetry breaking to $N = 6, 4, 2, 0$ in $D = 5$ may occur depending on how many mass parameters are taken different from zero.

2. Reduction of the (2,0) theory

We give here the qualitative features of the SS reduction of a general $(2,0)$ theory from $D = 6$ to $D = 5$ and show how it produces an $N = 2$ theory in $D = 5$ with tensor, vector and hyper multiplets, and a flat gauge group.

JHEP06(2004)018

Let us consider a $D = 6$ theory with n_T tensor multiplets, n_V vector multiplets and n_H hypermultiplets. These theories are anomalous unless the condition

$$n_H - n_V + 29\ n_T = 273 \tag{2.1}$$

is satisfied [23, 24].

It was shown in ref. [11] that when performing a standard dimensional reduction to $D = 5$ on an anomaly-free $(2,0)$ theory, we obtain a particular class of $N = 2$, $D = 5$ theories. After the reduction, the geometry of the hypermultiplets ($\mathcal{M}_Q$) remains unchanged. The scalar manifold of the vector and tensor multiplets has a real special geometry [25]. Let $\mathcal{M}_R$ be this manifold in $D = 5$ and $d = \dim \mathcal{M}_R$.

On general grounds, real-special geometry consists essentially on an embedding of $\mathcal{M}$ in a manifold of dimension $d+1$ through a cubic polynomial constraint

$$\mathcal{V} = d_{IJK} t^I t^J t^K = 1\,, \qquad I, J, K = 1, \dots d+1\,.$$

The metric induced by the embedding from the metric in the higher dimensional manifold a_{IJ},

$$a_{IJ} = -\frac{1}{2}\partial_I \partial_J \ln \mathcal{V}\,, \qquad g_{ij} = a_{IJ}\partial_i t^I \partial_j t^J|_{\mathcal{V}=1}\,, \qquad i,j = 1, \dots d\,. \tag{2.2}$$

In the following, we will denote $G_{IJ} = a_{IJ}|_{\mathcal{V}=1}$. When the $D = 5$ theory comes from a dimensional reduction from $D = 6$, $d = n_T + n_V + 1$ (the extra scalar coming from the metric), and the cubic polynomial takes the particular form

$$\mathcal{V} = 3\,(z\eta_{rs} b^r b^s + C_{rxy}\ b^r a^x a^y)\,; \qquad r = 0, 1, \cdots n_T\,; \qquad x = 1, \cdots n_V\,. \tag{2.3}$$

η_{rs} is the $(1, n_T)$ lorentzian metric related to the space $\mathrm{SO}(1, n_T)/\,\mathrm{SO}(n_T)$ (parametrized by b^r) in (1.1), $z = \sqrt{g_{55}} = e^\sigma$ is the Kaluza-Klein scalar and $a^x = A^x_6$ are the axions.

We now focus on the cases when $\mathrm{SO}(1, n_T)$ is a global symmetry. This demands the coupling C_{rxy} to be an invariant coupling in the sense explained in section 1. One could then introduce a SS phase in the CSA of $\mathrm{SO}(n_T)$. Some of the vector and tensor multiplets are charged under this generator, so they acquire mass. In the $D = 5$ interpretation the vectors gauge a non-abelian flat group, but their scalar partners give no contribution to the scalar potential, in agreement with the known results on $D = 5$ gauged supergravity [26, 27]. The gauging of flat groups in the context of $N = 2$ supergravity has not been considered in previous classifications [28]. These gaugings are always of no-scale type due to the particular structure of the critical points [29].

Finally, we want to note that to uplift (oxidate [30, 31]) to $D = 6$ a five dimensional $N = 2$ supergravity a necessary condition is that the cubic polynomial defining the real special geometry has the form (2.3). All the homogeneous spaces with real special geometry fall in this category. These spaces have been classified in refs. [15, 16, 17, 18]; they were denoted as $L(q, P, \dot{P})$ in ref. [18]. We explain here this notation. Let $q = n_T - 1$ and let $\mathcal{D}_{n_T}$ be the real dimension of an irreducible representation of $\mathrm{Spin}(1, n_T)$. For $n_T = 1, 5$ mod 8 there are two inequivalent real or pseudoreal (quaternionic) representations. Let

JHEP06(2004)018

P and $\dot{P}$ denote the number of copies of such representations ($\dot{P}=0$ for $n_T \neq 1,5$ mod 8). Then, $n_V=(P+\dot{P})\mathcal{D}_{n_T}$. The R-symmetry group of Spin$(1,n_T)$ in the representation $(P,\dot{P})$ is denoted by $\mathcal{S}_q(P,\dot{P})$ (see [18, table 3]).

When $\dot{P}=0$, the notation $L(q,P)=L(q,P,0)$ is used. The symmetric spaces [25] correspond to the particular cases $L(1,1)$, $L(2,1)$, $L(4,1)$, $L(8,1)$, $L(-1,P)$ and $L(0,P)$. We also have $L(q,0)=L(0,q)$. They are reported in ref. [18, table 2]. The examples of SS reductions reported in this paper will actually fall in this class.

2.1 Tensor multiplet sector

The $D=5$ theory obtained through an ordinary Kaluza-Klein dimensional reduction contains n_T+n_V+1 vector multiplets. This is because the (anti) self-duality condition in $D=6$

$$\partial_{[\mu}B^r_{\nu\rho]}=\pm\frac{1}{3!}\epsilon_{\mu\nu\rho\lambda\tau\sigma}\partial^\lambda B^{r|\tau\sigma} \qquad \mu,\nu=1,\dots 6 \tag{2.4}$$

tells us that in $D=5$ the two form $B^r_{\mu\nu}$ is dual to the vector $B^r_{\mu 6}$ ($\mu,\nu=1,\dots 5$).

We want now to perform a SS generalized dimensional reduction instead. Let $M^r{}_s=-M_s{}^r$ be the SS phase in the CSA of the global symmetry $\mathrm{SO}(n_T)\subset\mathrm{SO}(1,n_T)$. The form B^0 (of the gravitymultiplet) is inert under $\mathrm{SO}(n_T)$, so in the rest of this subsection the value $r=0$ is excluded and $r=1,\dots n_T$. The $D=6$ anti self-duality condition gives now

$$\begin{aligned}\partial_{[\mu}B^r_{\nu\rho]} &= \frac{1}{3!}\epsilon_{\mu\nu\rho\lambda\tau 6}\left(\partial^6 B^{r|\lambda\tau}+2\partial^\lambda B^{r|\tau 6}\right)\\ &= \frac{1}{3!}\epsilon_{\mu\nu\rho\lambda\tau}\left(M^r{}_s B^{s|\lambda\tau}+F^{r|\lambda\tau}\right), \qquad \mu,\nu=1,\dots 5\,.\end{aligned} \tag{2.5}$$

where $F^r_{\lambda\tau}=2\partial_{[\lambda}B^r_{\tau]6}$.

Equation (2.5) can be rewritten as a self-duality condition for a massive two-form in five dimensions [32]. Assume that the Cartan element M is invertible; then we can define

$$\hat{B}^r_{\mu\nu}=B^r_{\mu\nu}+(M^{-1})^r{}_s F^s_{\mu\nu}\,,$$

so

$$\partial_{[\mu}\hat{B}^r_{\nu\rho]}=M^r{}_s\frac{1}{3!}\epsilon_{\mu\nu\rho\lambda\tau}\hat{B}^{s|\lambda\tau},$$

that is,

$$d\hat{B}^r=M^r{}_s{}^*\hat{B}^s\,.$$

For n_T even, an element M with non zero eigenvalues $\pm i m_\ell\neq 0$ ($\ell=1,\dots n_T/2$) is invertible. Then we have $n_T/2$ complex massive two-forms. For n_T odd, the matrix M has at least one zero-eigenvalue. The corresponding antisymmetric tensor B^{r_0} is a gauge potential which can be dualized to a vector. If some other eigenvalue m_ℓ is zero, the same argument applies and there will be a couple of tensors (or one complex tensor) which can be dualized to vectors.

Summarizing, in the five dimensional theory there are $2n\leq n_T$, massive tensor multiplets (or n complex ones) and n_T-2n+1 abelian vector multiplets, one of them formed with the vector which is dual (after reduction to $D=5$) to the self dual tensor present in the $D=6$ graviton multiplet. This vector is a singlet of the global symmetry group.

JHEP06(2004)018

2.2 The scalar potential in $D = 5$

In this section we compute the scalar potential of the SS reduced theory.

The scalar potential comes from the kinetic term of the scalar fields [1]. The only scalars at $D = 6$ are in the tensor and hyper multiplets, which parametrize the manifold in (1.1). We denote by φ^i, $i = 1, \dots n_T$ the coordinates on $\mathrm{SO}(1, n_T)/\mathrm{SO}(n_T)$, and let

$$v^a = v_i^a \partial_\mu \varphi^i dx^\mu = v_\mu^a dx^\mu\,, \qquad a = 1, \dots n_T$$

be the pull back to space time of the vielbein one form. Similarly, the quaternionic manifold $\mathcal{M}_Q$ [10, 33], with holonomy $\mathrm{SU}(2) \times \mathrm{USp}(2n_H)$, has coordinates q^u, $u = 1, \dots 4n_H$ and vielbein

$$\mathcal{U}^{\alpha A} = \mathcal{U}_u^{\alpha A} \partial_\mu q^u dx^\mu = \mathcal{U}_\mu^{\alpha A} dx^\mu\,, \qquad \alpha = 1, \dots, 2n_H\,, \qquad A = 1, 2\,.$$

There is still a scalar mode coming from the metric $e^\sigma = \sqrt{g_{55}}$.

For the scalar potential we obtain

$$V(\sigma, \varphi, q) = V_T^{SS} + V_H^{SS} = e^{-\frac{8}{3}\sigma} \left[v_6^a(\varphi) v_{6a}(\varphi) + \mathcal{U}_6^{\alpha A}(q)\, \mathcal{U}_6^{\beta B}(q) \mathbb{C}_{\alpha\beta} \epsilon_{AB} \right], \tag{2.6}$$

where $\mathbb{C}_{\alpha\beta}$ and ϵ_{AB} are the antisymmetric metrics.

We see that this potential is semipositive definite. The critical points occur at

$$v_6^a(\varphi) = 0 \qquad \text{and} \qquad \mathcal{U}_6^{\alpha A}(q) = 0\,, \tag{2.7}$$

so $V = 0$ at the critical points, which are then Minkowski vacua. The scalar σ is not fixed, so the theory is of no-scale type. Notice that (2.7) implies

$$v_6^a(\varphi) = v_i^a M_i^j \varphi^j = 0\,, \qquad \mathcal{U}_6^{\alpha A}(q) = \mathcal{U}_u^{\alpha A} M_v^u q^v = 0\,.$$

If the mass matrices have some vanishing eigenvalues, then this results in some moduli of the theory, other than σ. For n_T odd, since the tensor multiplet mass matrix has always one vanishing eigenvalue, there are at least two massless scalars. There are three massless vectors in this case.

The SS potential given in (2.6) should be compared to the most general gauging of $N = 2$, $D = 5$ supergravity [26, 27, 34]

$$V_{D=5} = V_T + V_H + V_R\,; \qquad V_T \geq 0\,; \quad V_H \geq 0\,,$$

where V_T and V_H are the contributions of tensor and hypermultiplets (separately positive) and V_R is the contribution from vector and gravity multiplets due to the quaternionic Killing prepotential P_I^X, $X = 1, 2, 3$ [33].

For a $D = 5$ gauging corresponding to a SS reduction, we then need $V_R = 0$.

The explicit form of V_R is [26, 27, 34]

$$V_R = -4 t_{IJ} G^{IK} G^{JL} P_K^X P_L^X = -\frac{4}{3} \left(\frac{1}{3} (t^{-1})^{IJ} + t^I t^J \right) P_I^X P_J^X\,, \tag{2.8}$$

where $t_{IJ} = d_{IJK} t^K$ and $G^{IJ} = a_{IJ}|_{\mathcal{V}=1} = -\frac{1}{3}(t^{-1})^{IJ} + t^I t^J$ (see (2.2)).

Even when there are no hypermultiplets, this term is not necessarily zero, because one can take a constant prepotential, $P_I^{X_0} = g_I$ =constant (the rest zero). g_I is the $N = 2$ Fayet-Iliopoulos parameter, and we retrieve the particular form of V_R found in ref. [20].

Equation (2.8) can also be written as

$$V_R = -4d^{IJK} t_I P_J^X P_K^X \,, \tag{2.9}$$

where indices are lowered and raised with the metric G_{IJ}. For symmetric spaces one has $d_{IJK} = d^{IJK}$ [20].

From the point of view of the SS reduction, the constant prepotential corresponds to an SU(2) phase, which in absence of hypermultiplets only gives masses to the fermions. Therefore we must have $V_R = 0$ for any value of t^I in the reduced theory. Moreover, since this depends only on the real special geometry (see (2.9)), this conclusion also holds in presence of hypermultiplets.

In the SS reduction the vector gauging the $\mathrm{U}(1) \subset \mathrm{SU}(2)$ is the partner of the scalar $z = e^\sigma$, so $P_z^X \neq 0$ and the rest are zero. $V_R = 0$ then requires

$$(t^{-1})^{zz} = -3(t^z)^2 \quad \Longleftrightarrow \quad G^{zz} = 2(t^z)^2 \,. \tag{2.10}$$

Let us consider some particular examples of theories with $V_R = 0$. Setting $n_V = 0$, equation (2.3) becomes

$$\mathcal{V} = 3z\eta_{rs} b^r b^s$$

and one can check that (2.10) holds [20]. It also holds for the spaces $L(0, P)$. More generally, it holds for all symmetric spaces with real special geometry because of the relations $d_{IJK} = d^{IJK}$ and $d_{zzI} = 0$. They readily imply $V_R = 0$.

We have checked that there are in fact counterexamples to the condition $V_R = 0$ among the theories classified in [17, 18] which are all of the form (2.3), so $V_R = 0$ is a further restriction satisfied by the $D = 5$ real geometries that can be uplifted (oxidated) to $D = 6$. It would be interesting to know, in the general case, what the conditions on the coefficients $C^r_{xy} = \eta^{rs} C_{sxy}$ in (2.3) are to have $V_R = 0$.

We will see in the next subsection that the possible resolution of this puzzle lies in the cancellation of anomalies of the six dimensional theory.

2.3 Conditions for uplifting $D = 5$ to $D = 6$ theories

In $D = 6$, $(2, 0)$ chiral theories it was found that there is, in general, a clash between the gauge invariance of the two-forms and the gauge invariance of the 1 forms (vector fields). For generic couplings C_{rxy}, in the abelian case, the n_V U(1) currents J_x are not conserved but satisfy the equation [11, 14]

$$d^*J_x = \eta_{rs} C^r_{xy} C^s_{zw} F^y \wedge F^z \wedge F^w \,. \tag{2.11}$$

This violation of the gauge invariance implies also a violation of supersymmetry because the theory is formulated in the Wess–Zumino gauge and the supersymmetry algebra closes only up to gauge transformations [11]. The current is conserved if the constants C^r_{xy} satisfy

$L(q,P)$	$L(1,1)$	$L(2,1)$	$L(4,1)$	$L(8,1)$
(n_T, n_V, n_H)	(2,2,217)	(3,4,190)	(5,8,136)	(9,16,28)

Table 1: Exceptional symmetric spaces.

the condition

$$\eta_{rs} C^r_{x(y} C^s_{zw)} = 0 \,. \tag{2.12}$$

This condition is equivalent to the seemingly stronger condition

$$\eta_{rs} C^r_{(xy} C^s_{zw)} = 0 \tag{2.13}$$

because $C^r_{xy} = C^r_{yx}$. This can also be seen from the fact that the anomaly polynomial [11]

$$A \sim \eta_{rs} C^r_{xy} C^s_{zw} F^x \wedge F^y \wedge F^z \wedge F^w$$

vanishes if (2.13) holds. It is interesting to observe in this respect that among the homogeneous spaces in ref. [17, 18] only the symmetric spaces, with the exception of the family $L(-1,P)$, $P > 0$, satisfy this condition [17, 18].

Also, we must note that the symmetric spaces satisfying (2.13) do have in fact $V_R = 0$, while for the homogeneous, non symmetric cases there are counterexamples.

Condition (2.12) is only required for a $D = 6$ ungauged supergravity. If the theory in $D = 6$ is already gauged, the terms in the right hand side of (2.11) may be compensated by (one loop) quantum anomalies through a Green-Schwarz mechanism, namely, the lagrangian becomes a Wess-Zumino term [9, 12, 14]. The $D = 6$ potential is semipositive definite and simply given by [8]

$$V_{D=6} \simeq \sqrt{g} P^X_x P^X_y (C^{-1})^{xy} \,, \qquad \text{where } C_{xy} = C_{rxy} b^r \,.$$

The $D = 6$ supersymmetric vacua occur at $P^X_x = 0$. An hypermultiplet can be "eaten" by a vector multiplet, making it massive. Note that there are not BPS particle multiplets in $D = 6$. The additional contribution to the potential in $D = 5$ is

$$\sqrt{g_5}\, e^{-\frac{2}{3}\sigma} P^X_x P^X_y (C^{-1})^{xy} \,.$$

Since in this case V_R needs not to vanish, one may find new vacua in the SS reduction.

As an illustration of spaces satisfying (2.13), we give the spectrum of tensor, vector and hypermultiplets for the exceptional symmetric spaces in table 1. Note that the values of n_T and n_V are given by the uplifting (oxidation) procedure of ref. [35, 31]. These spaces are contained in the classification of refs. [17, 18] and consequently have a cubic polynomial of the form (2.3). The number n_H instead is fixed by the gravitational anomaly cancellation (2.1). For generic SS phases, the $L(1,1)$ model has one massless scalar and two massless vectors. All the other exceptional models have two massless scalars and three massless vectors.

There are no other solutions in the series $L(q,P,\dot{P})$. It is obvious that for non homogeneous spaces the constants C^r_{xy} are rather arbitrary and there may be much more solutions to the uplifting condition.

JHEP06(2004)018

However, in order to have a SS phase in the tensor and vector multiplet sector, non homogeneous spaces should have at least a residual U(1) isometry.

We therefore find that a possible explanation to the fact that $V_R \neq 0$ in $D = 5$ supergravity with cubic form (2.3) may be connected to the violation of supersymmetry in the six dimensional theory.

3. SS reduction of the $N = (4,4)$, $D = 6$ theory

Let us sketch here the general features and mass spectrum of the $D = 5$ theory obtained by SS compactification from the maximal supergravity in $D = 6$.

The gravitational multiplet of the six dimensional theory contains the graviton e_μ^m, four gravitini ψ_A, $A = 1, \ldots 4$ in the fundamental of $\mathrm{Sp}(4)_L$, four gravitini $\psi_{\tilde{A}}$, $\tilde{A} = 1, \cdots 4$ in the fundamental of $\mathrm{Sp}(4)_R$, five self dual and five anti-self dual 2-form potentials B_+^r, $B_-^{\dot{r}}$, $r, \dot{r} = 1, \cdots 5$ in the fundamental of SO(5,5), 16 vector potentials A_μ^α, $\alpha = 1, \cdots 16$ in the spinorial of SO(5,5), 20 dilatini $\chi^{r\tilde{A}}$ in the $(\mathbf{5},\mathbf{4})$ of $\mathrm{USp}(4)_L \times \mathrm{USp}(4)_R$, 20 dilatini $\chi^{A\dot{r}}$ in the $(\mathbf{4},\mathbf{5})$ of $\mathrm{Sp}(4)_L \times \mathrm{Sp}(4)_R$, and 25 scalars $\varphi^{r\dot{s}}$ spanning the scalar manifold $\mathrm{SO}(5,5)/(\mathrm{SO}(5) \times \mathrm{SO}(5))$.

The global symmetry of the theory is the maximal compact subgroup of Spin(5,5), $\mathrm{USp}(4)_L \times \mathrm{USp}(4)_R$, so that one can turn on an SS phase in its CSA. Since the rank is 4, we have 4 mass parameters $m_i, \tilde{m}_\ell$ $(i, \ell = 1, 2)$.

In $D = 5$ we obtain a maximal supergravity gauged with the flat group $\mathrm{U}(1) \ltimes \mathbf{16}$. The U(1) factor in the CSA is gauged by the Kaluza-Klein graviphoton B_μ and the 16 translations are gauged by the vectors $Z_\mu^\alpha = A_\mu^\alpha - A_6^\alpha B_\mu$. This is a flat subgroup of $\mathrm{E}_{6(6)}$, according to the Lie algebra decomposition [22]

$$\mathfrak{e}_{6(6)} \rightarrow \mathfrak{so}(5,5) \oplus \mathfrak{so}(1,1) + \mathbf{16}^+ + \mathbf{16}^- ,$$

and it gives a gauging of $N = 8$, $D = 5$ supergravity not included in previous classifications [36].

For generic values of m_i, m_ℓ all the Z^α vector fields become massive through the Higgs mechanism, with masses $|m_i \pm \tilde{m}_\ell|$. This can be understood from the fact that the spinorial representation $(\mathbf{16})$ of SO(5,5) is the $(\mathbf{4},\mathbf{4})$ of $\mathrm{USp}(4)_L \times \mathrm{USp}(4)_R$, so that the Scherk-Schwarz phase in this sector is $M^A{}_B \otimes 1 + 1 \otimes M^{\tilde{A}}{}_{\tilde{B}}$, with eigenvalues $\pm \mathrm{i} m_i \pm \mathrm{i}\tilde{m}_\ell$. On the other hand, the Kaluza-Klein graviphoton B_μ stays massless. The 16 axions have been absorbed by the 16 vectors to become massive.

In the scalar sector the SS phase appears in the kinetic terms through

$$\partial_6 \varphi^{r\dot{r}} = M^r{}_s \varphi^{s\dot{r}} + \tilde{M}^{\dot{r}}{}_{\dot{s}} \varphi^{r\dot{s}} ,$$

where the antisymmetric matrices $M^r{}_s$, $\tilde{M}^{\dot{r}}{}_{\dot{s}}$ have eigenvalues $\pm\mathrm{i}(m_1 \pm m_2)$, 0 and $\pm\mathrm{i}(\tilde{m}_1 \pm \tilde{m}_2)$, 0 respectively. This builds up the (positive-definite) $D = 5$ scalar potential

$$V(\sigma, \varphi) = e^{-\frac{8}{3}\sigma} P_6^{r\dot{s}}(\varphi) P_{6r\dot{s}}(\varphi) . \tag{3.1}$$

which vanishes for $P_6^{r\dot{s}}(\varphi) = 0$.

From the $D = 5$ gauged supergravity point of view this corresponds to gauge an isometry in $\mathrm{E}_{6(6)}/\mathrm{USp}(8)$, with Killing vector

$$K = k^{r\dot{r}} \frac{\partial}{\partial \varphi^{r\dot{r}}} = (M^r{}_s \varphi^{s\dot{r}} + \tilde{M}^{\dot{r}}{}_{\dot{s}} \varphi^{r\dot{s}}) \frac{\partial}{\partial \varphi^{r\dot{r}}} .$$

We have that 24 out of the 25 scalars become massive, 16 of them with masses $| \pm m_1 \pm m_2 \pm \tilde{m}_1 \pm \tilde{m}_2|$, 4 of them with masses $| \pm m_1 \pm m_2|$, 4 of them with masses $| \pm \tilde{m}_1 \pm \tilde{m}_2|$, with only one massless scalar, other than the Kaluza-Klein scalar $\sqrt{g_{55}} = e^{\sigma}$.

As far as the antisymmetric tensors $(B^r_+, B^{\dot{r}}_-)$ are concerned, the vector representation **10** of $\mathrm{SO}(5,5)$ decomposes under the maximal compact subgroup

$$\mathbf{10} \to (\mathbf{5},\mathbf{1}) + (\mathbf{1},\mathbf{5}) .$$

The SS phase of this sector is

$$M^r{}_s \oplus \mathbf{0}_{5\times 5} + \mathbf{0}_{5\times 5} \oplus \tilde{M}^{\dot{r}}{}_{\dot{s}} .$$

Correspondingly, we have in $D = 5$ four complex antisymmetric tensors, two with masses $|m_1 \pm m_2|$ and two with masses $|\tilde{m}_1 \pm \tilde{m}_2|$, plus 2 massless tensors which may be dualized to abelian vectors. They do not participate in the gauging so they stay massless.

All the gravitini $\psi_A, \psi_{\tilde{A}}$ become massive, with masses (equal in couples) $|m_i|, |\tilde{m}_\ell|$ respectively.

The dilatini $\chi^{r\tilde{A}}, \chi^{A\dot{r}}$ also get masses. 16 of them have masses $| \pm m_1 \pm m_2 \pm \tilde{m}_\ell|$, 16 have masses $| \pm m_i \pm \tilde{m}_1 \pm \tilde{m}_2|$, 4 have masses $| \pm \tilde{m}_\ell|$ and 4 have masses $| \pm m_i|$.

We observe that the moduli space of this theory contains two scalars and locally is $\mathrm{SO}(1,1) \times \mathrm{SO}(1,1)$. If we set to zero one of the mass parameters (e.g. $m_1 = 0$) we get an unbroken $N = 2$, $D = 5$ supergravity with two massless vector multiplets.

There are two ways of getting $N = 4$ supersymmetry, either we set $m_1 = m_2 = 0$ or $m_1 = \tilde{m}_1 = 0$. Finally, setting $m_1 = m_2 = \tilde{m}_1 = 0$ we get an $N = 6$ theory [37].

Acknowledgments

M.A. Ll. wants to thank the Physics and Mathematics Departments at UCLA for their hospitality during the realization of this work.

The work of S.F. has been supported in part by the D.O.E. grant DE-FG03-91ER40662, Task C, and in part by the European Community's Human Potential Program under contract HPRN-CT-2000-00131 Quantum Space-Time, in association with INFN Frascati National Laboratories.

The work of M.A. Ll. has been supported by the research grant BFM 2002-03681 from the Ministerio de Ciencia y Tecnología (Spain) and from EU FEDER funds and by D.O.E. grant DE-FG03-91ER40662, Task C.

S.F. wants to thank C. Angelantonj and specially A. Sagnotti for illuminating discussions.

References

[1] J. Scherk and J.H. Schwarz, *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61;
E. Cremmer, J. Scherk and J.H. Schwarz, *Spontaneously broken $N = 8$ supergravity*, *Phys. Lett.* **B 84** (1979) 83.

[2] L. Andrianopoli, R. D'Auria, S. Ferrara and M.A. Lledo, *Gauging of flat groups in four dimensional supergravity*, *J. High Energy Phys.* **07** (2002) 010 [hep-th/0203206].

[3] L. Andrianopoli, R. D'Auria, S. Ferrara and M.A. Lledo, *Duality and spontaneously broken supergravity in flat backgrounds*, *Nucl. Phys.* **B 640** (2002) 63 [hep-th/0204145].

[4] A. Dabholkar and C. Hull, *Duality twists, orbifolds, and fluxes*, *J. High Energy Phys.* **0309** (2003) 054 [hep-th/0210209].

[5] B. de Wit, H. Samtleben and M. Trigiante, *On lagrangians and gaugings of maximal supergravities*, *Nucl. Phys.* **B 655** (2003) 93 [hep-th/0212239].

[6] For a comprehensive review, see e.g. A.R. Frey, *Warped strings: self-dual flux and contemporary compactifications*, hep-th/0308156.

[7] L.J. Romans, *Selfduality for interacting fields: covariant field equations for six-dimensional chiral supergravities*, *Nucl. Phys.* **B 276** (1986) 71.

[8] H. Nishino and E. Sezgin, *The complete $N = 2$, $D = 6$ supergravity with matter and Yang-Mills couplings*, *Nucl. Phys.* **B 278** (1986) 353.

[9] A. Sagnotti, *A note on the Green-Schwarz mechanism in open string theories*, *Phys. Lett.* **B 294** (1992) 196 [hep-th/9210127].

[10] J. Bagger and E. Witten, *Matter couplings in $N = 2$ supergravity*, *Nucl. Phys.* **B 222** (1983) 1.

[11] S. Ferrara, R. Minasian and A. Sagnotti, *Low-energy analysis of M and F theories on Calabi-Yau threefolds*, *Nucl. Phys.* **B 474** (1996) 323 [hep-th/9604097].

[12] S. Ferrara, F. Riccioni and A. Sagnotti, *Tensor and vector multiplets in six-dimensional supergravity*, *Nucl. Phys.* **B 519** (1998) 115 [hep-th/9711059].

[13] F. Riccioni and A. Sagnotti, *Consistent and covariant anomalies in six-dimensional supergravity*, *Phys. Lett.* **B 436** (1998) 298 [hep-th/9806129].

[14] H. Nishino and E. Sezgin, *New couplings of six-dimensional supergravity*, *Nucl. Phys.* **B 505** (1997) 497 [hep-th/9703075].

[15] D.V. Alekseevskii, *Classification of quaternionic spaces with a transitive solvable group of motions*, *Math. USSR Izvestija* **9** (1975) 297.

[16] S. Cecotti, *Homogeneous Kahler manifolds and T algebras in $N = 2$ supergravity and superstrings*, *Commun. Math. Phys.* **124** (1989) 23.

[17] B. de Wit and A. Van Proeyen, *Special geometry, cubic polynomials and homogeneous quaternionic spaces*, *Commun. Math. Phys.* **149** (1992) 307 [hep-th/9112027].

[18] B. de Wit, F. Vanderseypen and A. Van Proeyen, *Symmetry structure of special geometries*, *Nucl. Phys.* **B 400** (1993) 463 [hep-th/9210068].

JHEP06(2004)018

[19] L. Andrianopoli, S. Ferrara and M.A. Lledo, *Scherk-Schwarz reduction of $D = 5$ special and quaternionic geometry*, hep-th/0405164.

[20] M. Günaydin, G. Sierra and P.K. Townsend, *Vanishing potentials in gauged $N = 2$ supergravity: an application of Jordan algebras*, *Phys. Lett.* **B 144** (1984) 41.

[21] *Supergravities in diverse dimensions*, vol. 1, 2, A. Salam and E. Sezgin eds., Singapore: World Scientific, Amsterdam, Netherlands, North-Holland, 1989, p. 1499.

[22] B. de Wit, H. Samtleben and M. Trigiante, *Gauging maximal supergravities*, *Fortschr. Phys.* **52** (2004) 489 [hep-th/0311225].

[23] S. Randjbar-Daemi, A. Salam, E. Sezgin and J. Strathdee, *An anomaly free model in six-dimensions*, *Phys. Lett.* **B 151** (1985) 351.

[24] A. Salam and E. Sezgin, *Anomaly freedom in chiral supergravities*, *Phys. Scripta* **32** (1985) 283.

[25] M. Günaydin, G. Sierra and P.K. Townsend, *Exceptional supergravity theories and the magic square*, *Phys. Lett.* **B 133** (1983) 72; *The geometry of $N = 2$ Maxwell-Einstein supergravity and jordan algebras*, *Nucl. Phys.* **B 242** (1984) 244.

[26] M. Günaydin and M. Zagermann, *The gauging of five-dimensional, $N = 2$ Maxwell-einstein supergravity theories coupled to tensor multiplets*, *Nucl. Phys.* **B 572** (2000) 131 [hep-th/9912027]; *Gauging the full R-symmetry group in five-dimensional, $N = 2$ Yang-Mills/Einstein/tensor supergravity*, *Phys. Rev.* **D 63** (2001) 064023 [hep-th/0004117]; *Unified Maxwell-Einstein and Yang-Mills-Einstein supergravity theories in five dimensions*, *J. High Energy Phys.* **07** (2003) 023 [hep-th/0304109].

[27] A. Ceresole and G. Dall'Agata, *General matter coupled $N = 2$, $D = 5$ gauged supergravity*, *Nucl. Phys.* **B 585** (2000) 143 [hep-th/0004111].

[28] J.R. Ellis, M. Günaydin and M. Zagermann, *Options for gauge groups in five-dimensional supergravity*, *J. High Energy Phys.* **11** (2001) 024 [hep-th/0108094].

[29] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, *Naturally vanishing cosmological constant in $N = 1$ supergravity*, *Phys. Lett.* **B 133** (1983) 61;
J.R. Ellis, A.B. Lahanas, D.V. Nanopoulos and K. Tamvakis, *No-scale supersymmetric standard model*, *Phys. Lett.* **B 134** (1984) 429;
A.B. Lahanas and D.V. Nanopoulos, *The road to no scale supergravity*, *Phys. Rept.* **145** (1987) 1.

[30] E. Cremmer, B. Julia, H. Lu and C.N. Pope, *Higher-dimensional origin of $D = 3$ coset symmetries*, hep-th/9909099.

[31] A. Keurentjes, *The group theory of oxidation*, *Nucl. Phys.* **B 658** (2003) 303 [hep-th/0210178]; *The group theory of oxidation, II. Cosets of non-split groups*, *Nucl. Phys.* **B 658** (2003) 348 [hep-th/0212024]; *Oxidation = group theory*, *Class. Quant. Grav. Class. and Quant. Grav.* **20** (2003) S525.

[32] P.K. Townsend, K. Pilch and P. van Nieuwenhuizen, *Selfduality in odd dimensions*, *Phys. Lett.* **B 136** (1984) 38.

[33] L. Andrianopoli et al., *$N = 2$ supergravity and $N = 2$ super Yang-Mills theory on general scalar manifolds: symplectic covariance, gaugings and the momentum map*, *J. Geom. Phys.* **23** (1997) 111 [hep-th/9605032].

JHEP06(2004)018

[34] E. Bergshoeff et al., *$N = 2$ supergravity in five dimensions revisited*, *Class. and Quant. Grav.* **21** (2004) 3015 [hep-th/0403045].

[35] B. Julia, *Superspace and supergravity. proceedings, Nuffield workshop, Cambridge, U.K., June 16–July 12, 1980*, S.W. Hawking and M. Rocek eds., Cambridge, U.K. Univ. Press, Cambridge 1981.

[36] L. Andrianopoli, F. Cordaro, P. Fré and L. Gualtieri, *Non-semisimple gaugings of $D = 5$ $N = 8$ supergravity and fdas*, *Class. and Quant. Grav.* **18** (2001) 395 [hep-th/0009048].

[37] E. Cremmer, *Superspace and supergravity. proceedings, Nuffield workshop, Cambridge, U.K., June 16–July 12, 1980*, S.W. Hawking and M. Rocek eds., Cambridge, U.K. Univ. Press, Cambridge 1981.